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Decision Criteria for the Deployment of Distributed Generation Technologies

Abstract

The recent emergence of distributed generation in the competitive electric power market raises many interesting issues. The deployment of distributed generation technologies required a decision tool in order to justify the best technological choice. This paper introduces decision criteria from the perspectives of distributed generation developers and end-use customers. The formulations include independent and coupled decisions. Load-priority method is discussed for the calculation of the load-point reliability.

1 Introduction

One of the interesting developments of the electric power industry has been the emergence of the distributed generation. It has changed the landscape of the competitive electric power market technically and economically. Unlike large-scale power plants, is distributed generation located in distribution power systems and mostly provides services in retail markets. Therefore, it will be a driving force to promote retail competition.

Although distributed resources have been used by regulated utilities for a long time, distributed generation in the new industry structure serves completely different purposes. In the regulated electric power industry, distributed generation has been used by utility distribution companies as a resource for deferral of the investment in T&D systems. However, it has not been used by non-utility entities for commercial purposed. Due to the development of the competitive power market and other factors, distributed generation has become a viable alternative to conventional power generation. As noted by Blazwicz, S., and Kleinschmidt, D. (1999c) [2], the recent emergence of distributed generation is a result of three independent trends--electric power industry restructuring, increasing system capacity needs, and technology advancements—that are currently laying the ground work for its possible widespread introduction.

Distributed generation can be used for many applications, such as, generating power, providing standby service or reserve capacity, shaving peak demand, providing ancillary services, and serving as stand-alone generation and cogeneration. Given the potential applications, distributed generation provides several benefits to both developers[1] and end-use customers. Developers can use distribution generation to lower costs of services, provide additional services and decrease exposure to electricity price volatility. Having consumption choices, end-use customer can reduce energy expenditures, increase reliability and power quality, reduce fuel costs for other energy needs, and receive a new source of revenues from electricity sales to the grid.

The entry of distributed generation technologies confronts a number of challenges. As noted by Cardell, J.B. (1997) [3], the integration of distributed generation into the distribution system requires a consideration of engineering, economic and policy issues represented by technical integration, market integration, and policy interactions. The complexity of distributed generation integration posts a challenging question. How should the decisions to deploy distributed generation technologies be made? Centralized integrated resource planning is certainly not applicable since the decisions are made independently. Different market participants have different decision criteria. Consumer choices introduce a complex decision task to end-use customers. The developers need to make decisions about their technology choices. This paper describes the decision criteria of developers and end-use customers. The objective is to provide an analytical tool for the decision-making for the deployment of distributed generation technologies.

2 Decision Making

In a competitive market, there is no centralized planning. Market participants, acting as economic agents, make decisions independently. The decisions are made based on economic criteria, i.e. maximizing benefits or minimizing costs, whichever is appropriate. Unlike wholesale markets, in which there are institutes, for example the ISO and Power Exchange, performing the technical and market coordination, there is no market coordinator in the distributed power market. Therefore, transactions are mainly based on bilateral deals between

[1] In this paper, a distributed generation developer is generally defined as an entity that deploys distributed generation for commercial purposes, for example energy service provider, load service entity, and independent investor.

buyers and sellers. Prices and other provisions in contracts, such as the penalty for loss of supply, are the results of bilateral agreement.

In the following three sections, decision criteria from the perspectives of distributed generation developers and end-use customers are developed. Sections 3 and 4 define the decision criteria from an individual decision maker's perspective. In other words, distributed generation developers and end-use customers make decisions independently. Realistically, in order to get a deal, distributed generation developers need to convince end-use customers by presenting the benefits of distributed generation. If the distributed generation provides a large amount of benefits, the developers can easily get the deal done. Therefore, the more appropriate approach is to incorporate customers' benefits into the decision-making process of the developers. Section 5 introduces coupled decision criteria from the perspective of distributed generation developers by including customers' benefits in the objective function.

It is worth noting that the approach proposed here is not a stand alone analytical method. It should be used as a supplement to other financial/economic analyses, which incorporate more financial and economic information, such as the costs of capital and inflation. The proposed approach is used to find the optimal technology choice after the investment is considered feasible.

The problem that we are interested in is the decisions to deploy distributed generation technologies in a competitive retail market. There are two possible cases.

<u>Case 1</u>

A developer approaches potential customers to sell distributed generation services. The decisions of the developer are the optimal technology choice as well as other decision variables, for example, installed capacity, the reliability of the generator, generation output and power purchase from the market.

<u>Case 2</u>

In contrast to customers in the monopoly industry, the customers in the competitive market have consumption choices. The customers need to consider the potential benefits of different choices, for example sticking with the existing utility, switching to distributed-generation suppliers, buying reserve power from distributed generation, investing in self-generation, etc.

2.1 Assumptions

The decision criteria are developed based on the following assumptions:

1. All market participants are rational and make decisions based on economic criteria.
2. Transactions between sellers and buyers are based on bilateral deals. Prices of products and services are the results of bilateral agreement.
3. Reliability[1] is measurable and constant through out the analysis time frame.
4. There are not startup and shutdown costs for the distributed generators.
5. Startup and shutdown times of the distributed generators are neglected.
6. Interest rates and costs of capital are constant.

2.2 Symbols

Throughout this chapter, the following symbols are used.

E [*] = Expected value of *

PV [*] = Present value of *

f (*) = Probability density function of *

Π = Net present value of developer's profit

σ_Π = Volatility or standard deviation of profit[1]

Ω = Net present value of customers' surplus

σ_Ω = Volatility of consumer's surplus

r = Risk factor

ρ = Discount rate

β = Probability that the reserve power is called for service

τ = Parameter of technology choice

κ = Parameter of consumption choice

T = Analysis period (e.g. one year)

R_i = Load-point reliability

R_j = Supply-point reliability

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[1] In this paper, reliability is defined as a fraction of the number of hours that the generation is in service in a year. R=h/8760, where h is total of hours that the generator is in service.

R_G = Reliability of the distributed generation

R_U = Reliability of the utility supply

R_W = Reliability of the wire

R_M = Reliability of the market supply

Q^D = Load demand (kW)

Q^G = Generation output (kW)

Q^S = Total supply power (kW)

Q_P = Real power (kW)

Q_R = Reserve power (kW)

Q^M = Quantity supplied from the market (kW)

Q^D_{min} = Minimum demand limited by minimum electricity-related activities (kW)

Q^D_{max} = Maximum demand limited by the installed capacity of the electrical load (kW)

Q^G_{min} = Minimum generation (kW)

$E[En^D]$ = Expected energy requirement (kWh)

K^G = Installed generation capacity (kW)

Q (p) = Customer demand function

P (Q) = Inverse demand curve of the customer

p, p_{DG} = Real power price of the distributed generation ($/kWh)

p_U = Real power price of the utility ($/kWh)

p_M = Market real power price ($/kWh)

p_R = Reserve power price ($/kWh)

p_L = Compensation received from the backup power supplier for the lost load ($/kWh)

p_{BU} = Backup power price of the utility ($/kWh)

c ( ) = Generation cost ($/kWh)

c_{LS} = Cost of lost supply ($/kWh)

c_{LL} = Customers' cost of lost load ($/kWh)

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[1] In this paper, volatility and standard deviation of profit are substitutable. Volatility is calculated from the square root of variance.

C ( ) = Annualized capital cost ($)

3 Decision Criteria of Distributed Generation Developers

The roles of the distributed generation developers are looking for investment opportunities and deciding to invest in profitable projects. In order to do so, developers need to convince potential customers of the benefits of distributed generation. The selling points of distributed generation are, but not limited to, low price energy and high reliability compared to utilities' services. If the project is convincing, a purchase contract between the developer, as a seller, and the customer, as a buyer, is signed. The purchase contract can be as short as one year and as long as twenty years.

The developers can fulfill the obligations in the contract by providing the services from distributed generation or buying power from the market, whichever is cheaper. In addition, if the cost of generation and market price are higher than the compensation made to the customers, the developer can opt to stop the services and pay compensation. For the investment choices, the developers can choose a single generation technology or a combination of technologies, which is called a generation portfolio. For example, microturbines and fuel cells are installed to supply peak and base loads respectively. With regard to customers, the developers can choose one large customer or a group of customers (syndicate) for one project. Given the number of choices, the decision criteria for the developer are complicated.

3.1 Decision Criteria

The developer's objective is to maximize returns from an investment. The returns can be measured in terms of present value of profit. In a competitive market, there are many uncertainties that affect the developer's profit. The load demand and market price are major sources of uncertainty. Therefore, the profit cannot be calculated with certainty. What the developer can do is to use the expected value of profit to make investment decisions.

A developer is an economic agent who is rational and varies in risk profile. Some developers are risk takers, while others are risk averse. The risk-averse developer prefers the investment with low profit volatility, while the risk-taker can absorb a higher risk for higher returns. Therefore, the decision criteria of the developers include not only the expected value of profit, but also the risk profile. As described by Ilic and Skantze (2000) [5], the generalized form of the objective function can be written as:

$$\underset{Q,K,R,\tau}{\text{Max}}\ J = E[\Pi] - r\sigma_\Pi \tag{1}$$

The risk profile of the developer is reflected in the r factor. The risk-averse developer puts a higher r factor into the objective function to compensate for the risk in the form of profit volatility.

The present value of profit is given by:

$$\Pi = \sum_t \begin{bmatrix} PV[\text{Revenue}_t] \\ -PV[\text{Generation cost}_t] \\ -PV[\text{Payment for power purchased from market}_t] \\ -PV[\text{Cost of lost supply}_t] \end{bmatrix} - \text{Annualized Capital Cost} \tag{2}$$

The developer has revenue and cost streams over times. The revenue received at any time is a product of price and supply quantity. There are three cost components: generation cost, payment for power purchased form the market, and the cost of lost supply. The generation cost is a direct function of the generation output. When the market price is lower than the generation cost, the developer will purchase the power from the market to supply the load. In this case, there is no generation cost but the developer has to make a payment to the supplier. The cost of lost supply incurs when the power supply is not sufficient or completely unavailable. The developer can opt to pay the customer the penalty or buy backup service from other sources, such as utility distribution companies or energy service providers. In either case, the developer's cost is represented by the cost of lost supply.

The expected value and volatility of profit are written as:

$$E[\Pi] = \int_0^1 \Pi \, df(\Pi) \tag{3}$$

$$\sigma_\Pi = \sqrt{E[\Pi^2] - E^2[\Pi]} \tag{4}$$

The optimization is subject to three constraints. First, total supply equals generation output plus the quantity purchased from the market. Second, the total supply at any given time shall not be greater than the load demand. This is an inequality constraint because the developer can choose to undersupply the load if the cost of supply is higher than the compensation. Third, generation output is bound by the minimum generation (lower bound) and the installed capacity (upper bound). In conclusion, the decision criteria of the developer are based on solving the following optimization problem:

$$\begin{array}{l} \text{Objective Function :} \\ \underset{Q,K,R,\tau}{\text{Max}}\ J = E[\Pi] - rs_{\Pi} \\ \\ \text{Subject to :} \\ Q^{G}(t) + Q^{M}(t) = Q^{S}(t) \\ Q^{S}(t) \le Q^{D}(t) \\ Q^{G}_{min} \le Q^{G}(t) \le K^{G} \end{array} \tag{5}$$

In a particular situation, the developer has a number of technology choices (τ) that can serve the purposes of investment. The technical and cost characteristics of the technologies are varied. In any case, the capital cost of the distributed generation is an increasing function of capacity and reliability. For each technology choice, the developer chooses the optimal capacity (K^*) and optimal reliability (R^*). After the developer gets the optimal solutions for all technology choices, the optimal technology choice (τ^*) is decided from the overall objective function as the following equation:

$$\underset{\tau}{Max}\left(J_1, J_2, ..., J_{\tau}, ..., J_{n_{\tau}}\right) \tag{6}$$

There are a number of scenarios that distributed generation can be invested. The developer may install distributed generation to sell real power, reserve power or a combination of both to a customer or a group of customers. In addition, the developer can provide services from a generation portfolio. In the following sections, we discuss how the profit, presented in the objective function of Equation 5, is formulated in each scenario. It should be noted that the list is not exhaustive because it is not possible to cover all the scenarios that the developer could face. The objective is to present the concept of the approach, which can be applied under any circumstances.

3.2 Single Technology Investment Scenarios

3.2.1 Scenario 1: Selling real power to a customer

In the first scenario, the developer invests in distributed generation to sell only real power to a customer. In this case, the developer competes with other suppliers in the electric power market. The real power price that the developer can set should provide better benefits than those the customer is receiving from the existing supplier. The benefits do not come with the price only. Reliability improvement can reduce the cost associated with lost load and therefore increase benefits. Therefore, the price is set according to the reliability level and compensation for the loss of supply. It is a package deal, defined by $\{p, R, c_{LS}\}$. The customer makes a comparison with the package, $\{p, R, c_{LL}\}$, received from the existing supplier, in most cases the utility distribution company. In a competitive retail market, the utility price is the spot price plus applicable transmission and distribution charges.

In this section, it is assumed that the developer makes decisions independently without considering the customers' benefits. The customers' benefits will be incorporated into the decision criteria of the developer later in Section 5. For the developer, the present value of profit is given by:

$$\Pi = \int_{t=0}^{T} e^{-\rho t} \begin{bmatrix} p(t)\left(Q^{G}(t)R_{G} + Q^{M}(t)R_{M}\right) \\ - c\left(Q^{G}(t), \tau\right)R_{G} \\ - p_{M}(t)Q^{M}(t)R_{M} \\ - c_{LS}(t)\left[\left(Q^{D}(t) - Q^{S}(t)\right) + Q^{G}(t)\left(1 - R_{G}\right) + Q^{M}(t)\left(1 - R_{M}\right)\right] \end{bmatrix} dt - C\left(K^{G}, R_{G}, \tau\right) \qquad (7)$$

The revenue stream consists of two components. The customer pays the real power price (p) for the total power supplied by the developer. The power supplied from the distributed generator has reliability R_G, and therefore the payment is factored by this reliability fraction. Similarly, the payment received for the market power is factored by R_M. As for the costs, the developer incurs three variable cost components according to the three power portions. First, the developer pays the market price (p_M) for the power purchased from the market factored by its reliability fraction. The developer pays the customer the cost of lost supply in case that the power supplied from the market fails. Second, the generation cost of the distributed generation is factored by the generator reliability (R_G). When the generator fails and becomes unavailable, the developer pays the cost of lost supply to the customer. Third, the developer pays the cost of lost supply for the unmet demand. The last term of Equation 7 is a fixed cost from the capital investment.

3.2.2 Scenario 2: Selling reserve power to a customer

In this scenario, the developer installs distributed generation to provide reserve power[1] only. In the competitive market, the developer competes with other suppliers in the reserve market. Reserve power can be considered another commodity in the electric power market. The prices of reserve power are determined by market demand and supply. In a bilateral deal, the developer negotiates with the customer to set prices that satisfy both parties.

As discussed in Allen and Ilic (1999) [1], reserve power can be sold under two pricing schemes: (1) the power-delivered payment method and (2) the power-allocated payment method. The net present values of profits for the developer in both cases are defined in the following sections.

a. The Power-delivered Payment Method

In this method, the seller of the reserve power is paid the reserve power price only if the reserve power is actually used. There is no other payment made for the standby service. Since the excess power during the standby period does not generate revenue, the seller receives revenue during a short time period. Therefore, the reserve price is higher than the real power price. The present value of the developer's profit in this case can be written as:

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[1] In this paper, reserve power is synonymous with backup power.

$$\Pi = \int_{t=0}^{T} e^{-\rho t}\beta\begin{bmatrix} p_R(t)Q^G(t)R_G \\ -c\left(Q^G(t),\tau\right)R_G \\ -c_{LS}(t)\left[\left(Q^D(t)-Q^G(t)\right)+Q^G(t)\left(1-R_G\right)\right]\end{bmatrix}dt - C\left(K^G, R_G, \tau\right) \tag{8}$$

In this case, it is assumed that the developer does not buy power from the market because reserve power suppliers usually do not buy power from another supplier. Since the revenue and costs occur only when the reserve power is called for service, the first term of Equation 8 is factored by the probability that the reserve power is called for service (β). The developer can opt to undersupply the reserve load in case the generation cost is higher than the penalty for loss of supply. However, this is unlikely because the penalty for the reserve load is normally higher than the generation cost.

b. <u>The Power-allocated Payment Method</u>

In the power-allocated method, the seller is paid the reserve power price for the capacity allocated during the period when the reserve power is not called for service. When the reserve power is used, the seller receives the real power price for the power generated. In this case, the reserve power price is lower than the real power price. The developer's present value of profit is given by:

$$\Pi = \int_{t=0}^{T} e^{-\rho t}\begin{bmatrix} p_R(t)Q^D(t)(1-\beta)+p(t)Q^G(t)\beta R_G \\ -c\left(Q^G(t),\tau\right)\beta R_G \\ -c_{LS}(t)\beta\left[\left(Q^D(t)-Q^G(t)\right)+Q^G(t)\left(1-R_G\right)\right]\end{bmatrix}dt - C\left(K^G, R_G, \tau\right) \tag{9}$$

There are two revenue terms in the equation. In the non-service period, the developer receives a payment of $p_R(t)Q^D(t)$ for the standby power. This payment is calculated from the demand quantity specified in the contract. The payment is factored by the non-service fraction (1-β). The payment of $p(t)Q^G(t)$ is paid for the actual power delivered in the service period. This payment is factored by the service fraction (β) and the reliability fraction (R_G). The generation cost occurs only when reserve power is used. It is factored by the service fraction and the reliability fraction. Penalty is imposed in the event that the reserve power is called for service but the generator is not available and the load is undersupplied.

3.2.3 Scenario 3: Selling real power and reserve power

This is a two-product scenario, in which the developer allocates a part of the generation capacity to a customer who buys only real power and the other part of the generation capacity to a customer who buys only reserve power. It does not make any sense for one customer to buy both real power and reserve power from a supplier with only one generating unit. This is because when the real power supply is out of service, the reserve power is not able to function. In the case that the developer provides the services from two different generation units to a single customer, it can be treated as a case of generation portfolio, which will be discussed later in section 3.3.

As discussed Scenario 2, reserve power can be sold with two payment methods. The net present values of the developer's profits in both cases are discussed in the following sections.

a. <u>Real Power and Reserve Power with the Power-delivered Payment Method</u>

In the case that the developer sells real power and reserve power using the power-delivered payment method, the present value of profit can be written as:

$$\Pi = \int_{t=0}^{T} e^{-\rho t} \begin{bmatrix} p(t)\left(Q_P^G(t)R_G + Q^M(t)R_M\right) + p_R(t)Q_R^G(t)\beta R_G \\ - c\left(Q_P^G(t),\tau\right)(1-\beta)R_G - c\left(\left(Q_P^G(t)+Q_R^G(t)\right),\tau\right)\beta R_G \\ - p_M(t)Q^M(t)R_M \\ - c_{LS,P}(t)\left[\left(Q_P^D(t) - Q_P^G(t) - Q^M(t)\right) + Q_P^G(t)(1-R_G) + Q^M(t)(1-R_M)\right] \\ - c_{LS,R}(t)\left[\left(Q_R^D(t) - Q_R^G(t)\right) + Q_R^G(t)(1-R_G)\right]\beta \end{bmatrix} dt - C\left(K^G, R_G, \tau\right) \tag{10}$$

The profit equation is getting more complicated. There are two sources of revenue, one from real power sales and the other from reserve power sales. The cost components also increase corresponding to the two types of customer: real power load and reserve power load. However, the concepts of revenue and cost in Sections 3.2.1 and 3.2.2 are still valid.

b. Real Power and Reserve Power with the Power-allocated Payment Method

For the power-allocated payment method, the developer's present value of profit can be written as:

$$\Pi = \int_{t=0}^{T} e^{-\rho t} \begin{bmatrix} p(t)\left(Q_P^G(t)R_G + Q^M(t)R_M\right) + p_R(t)Q_R^D(t)(1-\beta) + p(t)Q_R^G(t)\beta R_G \\ -c\left(Q_P^G(t),\tau\right)(1-\beta)R_G - c\left(\left(Q_P^G(t)+Q_R^G(t)\right),\tau\right)\beta R_G \\ -p_M(t)Q^M(t)R_M \\ -c_{LS,P}(t)\left[\left(Q_P^D(t)-Q_P^G(t)-Q^M(t)\right)+Q_P^G(t)(1-R_G)+Q^M(t)(1-R_M)\right] \\ -c_{LS,R}(t)\left[\left(Q_R^D(t)-Q_R^G(t)\right)+Q_R^G(t)(1-R_G)\right]\beta \end{bmatrix} dt - C\left(K^G, R_G, \tau\right) \qquad (11)$$

Equation 12 is different from Equation 11 only in the revenue terms, and this difference is due to the payment method.

3.3 Generation portfolio

In this scenario, the developer installs a generation portfolio to supply real power to a customer or a group of customers, for example a shopping complex, office building or industrial park. The formulation of the profit equation is complicated. The reliabilities on both the generation and load sides, defined as the supply-point reliability and load-point reliability respectively, have to be calculated. The complexity of the problem increases with the number of generator units in the portfolio. In addition, load information is required to determine load curtailments in the event of a supply shortage due to generator failure. This paper proposes the load priority method to calculate the load-point reliability as discussed in Section 6.

Considering a single product case, real power, there are two possible scenarios: a generation portfolio for one customer and for a group of customers.

3.3.1 Scenario 1: Generation portfolio for a single customer

The present value of profit of the developer can be written as:

$$\Pi = \int_{t=0}^{T} e^{-\rho t} \begin{bmatrix} p(t)\left(\sum_{j=1}^{n_G} \left[Q_j^G(t) R_j \right] + Q^M(t) R_M \right) \\ - \sum_{j=1}^{n_G} \left[c_j \left(Q^G(t), \tau \right) R_j \right] \\ - p_M(t) Q^M(t) R_M \\ - c_{LS}(t) \left[\left(Q^D(t) - Q^S(t) \right) + \sum_{j=1}^{n_G} \left[Q_j^G(t)(1 - R_j) \right] + Q^M(t)(1 - R_M) \right] \end{bmatrix} dt - \sum_{j=1}^{n_G} \left[C_j \left(K^G, R_G, \tau \right) \right] \tag{12}$$

The formulation approach is similar to Equation 7. The total power supplied to the customer is defined by:

$$Q^S(t) = \sum_{j=1}^{n_G} Q_j^G(t) + Q^M(t) \tag{13}$$

3.3.2 Generation portfolio for a group of customers

The developer's present value of profit is written as:

$$\Pi = \sum_{i=1}^{n_D} \int_{t=0}^{T} e^{-\rho t} \begin{bmatrix} p_i(t)\left(Q_i^S(t) R_i \right) \\ - c_{LS,i}(t) \begin{bmatrix} \left(Q_i^D(t) - Q_i^S(t) \right) \\ + Q_i^S(t)(1 - R_i) \end{bmatrix} \end{bmatrix} dt - \int_{t=0}^{T} e^{-\rho t} p_M(t) Q^M(t) R_M - \sum_{j=1}^{n_{Gj}} \begin{bmatrix} \int_{t=0}^{T} e^{-\rho t} c_j \left(Q_j^G(t), \tau \right) R_j \\ + C_j \left(K^G, R_G, \tau \right) \end{bmatrix} \tag{14}$$

The power purchased from the market is considered as an additional source of supply to the generation portfolio. The developer puts together all the generation resources and power purchased from the market and allocates the resources to individual customers. Q_i^S is the quantity of supply allocated to customer i with load-point reliability R_i.

4 Decision Criteria of End-use Customers

4.1 Decision Criteria

In a competition retail market, end-use customers are provided with consumption choices. They need to make complicated consumption decisions, which have never been made before. They can choose the supplier for each electric power service, such as real power and reserve power. This section discusses the decision criteria of the individual customer.

In making decisions about suppliers and services, the customer's objective is to maximize the benefits from the consumption of electric power, in terms of consumer's surplus, over time. For each possible consumption choice, the customer adjusts the consumption pattern (or behavior) to maximize the consumer's surplus. The objective function of the customer can be written as:

$$\underset{Q}{\text{Max}}\ H = E[\Omega] - r\sigma_{\Omega} \tag{15}$$

The net present value of the consumer's surplus is given by:

$$\Omega = \sum_{t} PV[\text{Benefit}_{t}] - \sum_{t} PV[\text{Cost}_{t}] \tag{16}$$

Similar to the discussion in Section 3.1 regarding the present value of the developer's profit, the net present value of the consumer's surplus is not deterministic. The expected value for the consumer's surplus is calculated from its probability density function. The expected value and volatility of the consumer's surplus are written as:

$$E[\Omega] = \int_{0}^{1} \Omega df(\Omega) \tag{17}$$

$$\sigma_{\Omega} = \sqrt{E[\Omega^{2}] - E^{2}[\Omega]} \tag{18}$$

The optimization is bound by two constraints. First, the customer does not have much flexibility in increasing and decreasing the consumption quantity. The flexibility is limited by a minimum consumption requirement and by maximum "nameplate" capacities of the electrical loads. Second, within a certain period, the customer has to do a certain number of electricity-related activities. For example, some activities can be moved from one hour to another hour, but not from one day to another day. Therefore, the total consumption in one period is equal to the energy requirement of that period. In summary, the optimization problem can be written as:

$$\begin{array}{l} \text{Objective Function :} \\ \underset{Q}{\text{Max}}\ H = E[\Omega] - rs_{\Omega} \\ \\ \text{Subject to :} \\ Q^D_{min} \le Q^D(t) \le Q^D_{max} \\ \int_{t=0}^{T} Q^D(t)dt = E[En^D] \end{array} \tag{19}$$

After the net present value of the consumer's surplus for each consumption choice is maximized, the customer selects the optimal consumption choice (κ^*) giving the highest net present value of consumer's surplus with the overall objective function given by:

$$\underset{\kappa}{\text{Max}}\left(H_1, H_2, ..., H_\kappa, ..., H_{n_\kappa}\right) \tag{20}$$

4.2 Load-point Reliability

When the customer purchases power from distributed generation, there are two possible supply sources: utility supply and distributed generation. The utility supply source delivers power through distribution wires. There are two possible connection arrangements, defined as pattern I and pattern II. Figure 1 shows connection pattern I of the customer. The utility supply is connected to the tie-in point by a section of wire (W_1). The reliability of W_1 is r_{12}. Assuming the reliability of the utility supply to be r_{11}, the combined reliability of the utility supply is R_1 (mathematically equal to $r_{11}*r_{12}$). The distributed generation is connected to the distributed system at the tie-in point. There is a small section of distribution wire (W_2) connecting the generator with the tie-in point. Assuming the reliability of the generator to be r_{21} and the reliability of W_2 to be r_{22}, the combined reliability of the generator at the tie-in point is R_2 (mathematically equal to $r_{21}*r_{22}$). There is another section of wire (W_3) connecting the tie-in point to the load connection point. The reliability of W_3 is R_3. In connection pattern II, as shown in Figure 2, the distributed generator is installed at the load connection point. In this case, there is no distributed wire between the distributed generator and the customer. The load-point reliability of the customer (R_i) in various scenarios can be calculated as shown in Table 1.

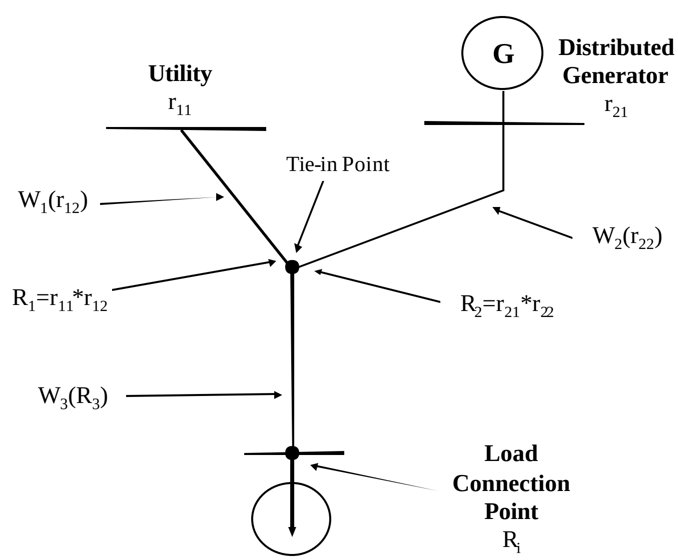

Figure 1: Connection Diagram of Distributed Generation Customer (Pattern I)

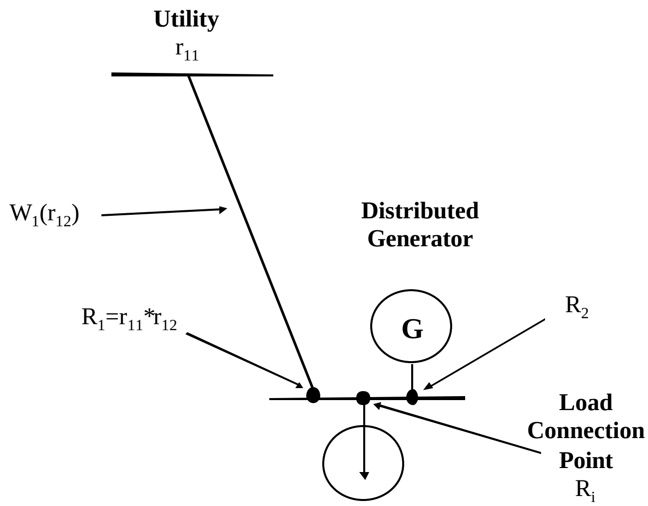

Figure 2: Connection Diagram of Distributed Generation Customer (Pattern II)

Table 1: Load-point Reliability Calculation

Supply Source	**Load-point Reliability (R_i)**	
	Pattern I	**Pattern II**
Utility only, R_U	R_1R_3	R_1
Distributed Generator only, R_G	R_2R_3	R_2
Both Utility and Distributed Generator	$(R_1+R_2-R_1R_2)R_3$	$(R_1+R_2-R_1R_2)$

4.3 Consumption Scenarios

The customer's consumption choices can be divided into two groups: short-term decisions and long-term decisions. In the short term, the customer chooses the best supplier and an optimal combination of products. The long-term decisions involve capital investments in distribution wires or owned-generation.

The following sections discuss the approach to the formulation of consumer's surplus equations in various scenarios. The formulation assumes that the customer connects to the distributed generator with connection pattern I. In case of connection pattern II, the reliability R_3 can simply be taken out. As stated in Section 3.1, the discussion cannot possibly cover all the situations that the customer might face. The intention is to discuss the concept of the formulation approach.

4.3.1 Scenario 1: Buying real power from the utility without backup power

This is a scenario in the traditional distributed power industry. The customer buys real power from the utility. The service is normally provided without backup. If the supply is interrupted, there is no reserve power provided. The customer is left without a power supply and unable to claim compensation from the utility. The customer incurs the cost of supply interruption in terms of the cost of lost load (c_{LL}). The net present value of the consumer's surplus is:

$$\Omega = \int_{t=0}^{T} e^{-\rho t}\left[\left(\int_{0}^{Q^D(t)} \mathbf{p}(Q)dQ\right)R_1R_3 - \left\{\begin{array}{l} p_U(t)Q^D(t)R_1R_3 \\ + c_{LL}(t)Q^D(t)(1-R_1R_3) \end{array}\right\}\right]dt \qquad (21)$$

The first term of Equation 21 is the customer utility from the consumption of real power (Q). The customer utility is factored by the supply reliability of the utility. The customer pays price p_U for the consumed power. The payment is factored by the reliability fraction of the supply. The last term is the cost of lost load. This cost occurs when there is a supply interruption.

4.3.2 Scenario 2: Buying real power from distributed generation without backup power

In this scenario, the customer switches real power supplier from the utility to distributed generation. The net present value of the consumer's surplus is written as:

$$\Omega = \int_{t=0}^{T} e^{-\rho t}\left[\left(\int_{0}^{Q^D(t)} p(Q)dQ\right)R_2R_3 - \left\{\begin{array}{l} p_{DG}(t)Q^D(t)R_2R_3 \\ + c_{LL}(t)Q^D(t)(1-R_2R_3) \end{array}\right\} + p_L(t)Q^D(t)(1-R_2R_3)\right]dt \qquad (22)$$

There is an additional term for the compensation received from the supplier. In contrast to the utility distribution company, the distributed generation supplier normally guarantees the quality of the supply by providing compensation (p_L) for a supply interruption. The compensation may or may not be equal to the cost of lost load. It depends on the negotiation between the supplier and the customer.

4.3.3 Scenario 3: Buying real power from the utility and backup power from distributed generation

This scenario adds one more supply source to Scenario 1. The real power is provided by the utility. Distributed generation provides backup service to increase reliability. As discussed in 3.2.2, the backup power can be purchased with two payment methods.

a. <u>The Power-delivered Payment Method</u>

With the power-delivered payment method, the net present value of the consumer's surplus is written as:

$$\Omega = \int_{t=0}^{T} e^{-\rho t} \left[\left(\int_{0}^{Q^D(t)} p(Q) dQ \right) R_i - \begin{Bmatrix} p_U(t) Q^D(t) R_1 R_3 \\ + p_{DG} Q^D(t)(1 - R_1) R_2 R_3 \\ + c_{LL}(t) Q^D(t)(1 - R_i) \end{Bmatrix} + p_L(t) Q^D(t)(1 - R_1)(1 - R_2 R_3) \right] dt \tag{23}$$

The customer's utility from the consumption of the power is presented by the first term of Equation 23. This term is factored by the load-point reliability of the customer as calculated in Table 1. The second term is the related costs of the consumption consisting of three parts: (1) payment to the utility for the consumed real power, (2) payment to the distributed generator for the used reserve power, and (3) the cost of lost load. The last term of the equation is the compensation received from the reserve power supplier, in this case the distributed generator.

b. <u>The Power-allocated Payment Method</u>

In the case of the power-allocated payment method, the net present value of the consumer's surplus is written as:

$$\Omega = \int_{t=0}^{T} e^{-\rho t}\left[\left(\int_{0}^{Q^D(t)} p(Q)dQ\right)R_i - \begin{Bmatrix} p_U(t)Q^D(t)R_1R_3 \\ + p_U(t)Q^D(t)(1-R_1)R_2R_3 \\ + p_{DG}(t)Q^D(t) \\ + c_{LL}(t)Q^D(t)(1-R_i) \end{Bmatrix} + p_L(t)Q^D(t)(1-R_1)(1-R_2R_3)\right]dt \tag{24}$$

Equation 24 is similar to Equation 23. The customer's utility and compensation for lost load are the same. The payment term is different due to the difference in the payment method.

4.3.4 Scenario 4: Buying real power from distributed generation and backup power from the utility

This scenario is similar to Scenario 3. The primary and backup supply sources are switched. The distributed generation supplies real power while the utility provides backup power. Again, the reserve power can be provided using either of the two payment methods. The net present values of the consumer's surplus for both methods are defined as the following:

a. <u>The Power-delivered Payment Method</u>

$$\Omega = \int_{t=0}^{T} e^{-\rho t}\left[\left(\int_{0}^{Q^D(t)} p(Q)dQ\right)R_i - \begin{Bmatrix} p_{DG}(t)Q^D(t)R_2R_3 \\ + p_{BU}Q^D(t)(1-R_2)R_1R_3 \\ + c_{LL}(t)Q^D(t)(1-R_i) \end{Bmatrix} + p_L(t)Q^D(t)(1-R_2)(1-R_1R_3)\right]dt \tag{25}$$

b. <u>The Power-allocated Payment Method</u>

$$\Omega = \int_{t=0}^{T} e^{-\rho t}\left[\left(\int_{0}^{Q^D(t)} p(Q)dQ\right)R_i - \begin{Bmatrix} p_{DG}(t)Q^D(t)R_2R_3 \\ + p_U(t)Q^D(t)(1-R_2)R_1R_3 \\ + p_{BU}(t)Q^D(t) \\ + c_{LL}(t)Q^D(t)(1-R_i) \end{Bmatrix} + p_L(t)Q^D(t)(1-R_2)(1-R_1R_3)\right]dt \tag{26}$$

4.3.5 Scenario 5: Buying real power from the utility and investing in wires

In this scenario, the customer musts make a long-term decision. This is the case of a new load, which has no connection to the distribution system. A distribution wire needs to be constructed if the customer wants to receive power from the distribution system. If the utility constructs this wire and charges a distribution charge embedded in the real power price, the problem formulation is the same as in Scenario 1. In this case, the utility does not invest in the wire. The customer has to invest in this piece of wire in order to connect to the distribution system. The net present value of the consumer's surplus is written as:

$$\Omega = \int_{t=0}^{T} e^{-\rho t}\left[\left(\int_{0}^{Q^D(t)} p(Q)dQ\right)R_U R_W - \left\{\begin{array}{l} p_U(t)Q^D(t)R_U R_W \\ + c_{LL}(t)Q^D(t)(1-R_U R_W)\end{array}\right\}\right]dt - C\left(K^W, R_W, D\right) \tag{27}$$

The customer's utility is factored by the load-point reliability, defined by the combined reliability of the utility supply and the wire ($R_U R_W$). The customer pays the utility price for the consumed real power. The cost of lost load occurs when the supply from the utility is interrupted with probability ($1-R_U R_W$). The last term is the capital cost of the wire.

4.3.6 Scenario 6: Investing in distributed generation

In this case, the customer decides to invest in distributed generation. The customer can be either a new load or an existing load that wants to be isolated from the distribution system. The net present value of the consumer's surplus is written as:

$$\Omega = \int_{t=0}^{T} e^{-\rho t}\left[\left(\int_{0}^{Q^D(t)} p(Q)dQ\right)R_G - \left\{\begin{array}{l} c\left(Q^D(t)\right)R_G \\ + c_{LL}(t)Q^D(t)(1-R_G)\end{array}\right\}\right]dt - C\left(K^G, R_G\right) \tag{28}$$

In Equation 28, the customer's utility is factored by the reliability of the distributed generation. There are two variable costs of consumption: generation cost and the cost of lost load. Each cost is factored by its probability of occurrence. The fixed cost is the annualized capital cost of distributed generation.

4.3.7 Scenario 7: Investing in distributed generation and buying backup power from the utility

In Scenario 6, the customer has only one supply source, which is distributed generation. In this next case, the customer may want to gain higher reliability by buying backup power from the utility. The backup power can be purchased with either of the two previously mentioned payment methods. The net present values of the consumer's surplus for the two payment methods are formulated below.

a. The Power-delivery Payment Method

$$\Omega = \int_{t=0}^{T} e^{-\rho t}\left[\left(\int_{0}^{Q^D(t)} p(Q)dQ\right)R_i - \begin{Bmatrix} c\left(Q^D(t)\right)R_G \\ + p_{BU}(t)Q^D(t)(1-R_G)R_U \\ + c_{LL}(t)Q^D(t)(1-R_i) \end{Bmatrix} + p_L Q^D(t)(1-R_G)R_U\right]dt - C\left(K^G, R_G\right) \tag{29}$$

b. The Power-allocated Payment Method

$$\Omega = \int_{t=0}^{T} e^{-\rho t}\left[\left(\int_{0}^{Q^D(t)} p(Q)dQ\right)R_i - \begin{Bmatrix} c\left(Q^D(t)\right)R_G \\ + p_{BU}(t)Q^D(t) \\ + p_U(t)Q^D(t)(1-R_G)R_U \\ + c_{LL}(t)Q^D(t)(1-R_i) \end{Bmatrix} + p_L Q^D(t)(1-R_G)R_U\right]dt - C\left(K^G, R_G\right) \tag{30}$$

5 Coupled Decision Criteria of the Developer

5.1 Concept

In Section 3, the decision criteria of the developer are independent from those of the customer. In other words, theoretically, the developer makes an investment decision without considering customer benefits. The developer's sole objective is to maximize the expected profit. However, practically, the developer needs to convince the customer to make a purchase agreement. The benefits of distributed generation need to be presented to attract the customer. The more benefit the customer gets, the more attractive the contract is. Therefore, the developer's objective is to maximize not only the expected profit, but also the customer's benefits.

In this section, coupled decision criteria are introduced with regard to the developer's decision about technology choice. The introduced method is applied to the situation in which a developer negotiates with a customer for a purchase contract. The developer can select from several technology choices, for example microturbines and fuel cells, to achieve the investment objective. The developer wants to choose the technology that maximizes the expected profit. However, the customer's benefits have to be maximized as well so that the purchase contract is attractive from the customer's perspective.

In order for an attractive investment to be designed, the decision criteria need to include the objective functions of both decision makers: the developer and the customer. Since the formulation of the problem involves two objective functions, it becomes a multi-objective optimization. Figure 3 shows how the decision criteria are coupled.

5.2 Technology Choice

This section discusses possible solutions to the developer's decision concerning the technology. For each technology choice, the developer optimizes the multi-objective functions to get a set of non-inferior solutions by using any of the solving techniques. Connecting the non-inferior solutions creates the Pareto optimal frontier. The Pareto optimal frontiers of all the possible technology choices are graphed in objective function space, represented by J (the expected developer profit) and H (the expected consumer's surplus) on the vertical axis and horizontal axis respectively. Using the curves in objective function space, the developer can choose the optimal technology for investment. Considering the example of two technology choices, technologies A and B, there are a number of possible solutions.

Solution 1

The Pareto optimal frontiers of the two technology choices are parallel, as shown in Figure 4. At any level of the expected consumer's surplus, technology A gives higher expected profits than technology B does. In this case, the optimal technology choice is technology A.

Developer's Decision Criteria

Objective Function :

$$\max_{K,R,\tau} J = E[\Pi] - rs_{\Pi}$$

Subject to :

$$Q^G(t) + Q^M(t) = Q^S(t)$$

$$Q^S(t) \le Q^D(t)$$

$$Q^G_{min} \le Q^G(t) \le K^G$$

Customer's Decision Criteria

Objective Function :

$$\max_{Q} H = E[\Omega] - rs_{\Omega}$$

Subject to :

$$Q^D_{min} \le Q^D(t) \le Q^D_{\mathbf{max}}$$

$$\int_{t=0}^{T} Q^D(t)dt = E[En^D]$$

Coupled Decision Criteria

Objective Functions :

$$\max_{K,R,\tau n Q} \mathbf{F}$$

Subject to :

$$\mathbf{G_D}(x) \le 0$$

$$\mathbf{G_{DE}}(x) = 0$$

$$\mathbf{G_C}(x) \le 0$$

$$\mathbf{G_{CE}}(x) = 0$$

Figure 3: Coupled Decision Criteria

Note:

$\mathbf{F}$ = vector of objective functions J and H

$\mathbf{G_D}$ = vector of inequality constraints of the developer

$\mathbf{G_{DE}}$ = vector of equality constraints of the developer

$\mathbf{G_C}$ = vector of inequality constraints of the customer

$\mathbf{G_{CE}}$ = vector of equality constraints of the customer

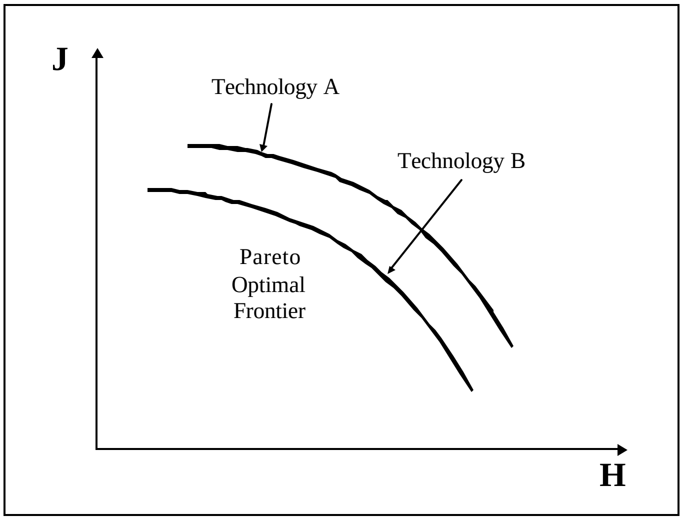

Figure 4: Parallel Pareto optimal frontiers

Solution 2

As shown in Figure 5, the Pareto optimal frontiers of the two technology choices intersect at one point. If the customer is likely to have high negotiation power and looking for a high-expected consumer's surplus (higher than H_0), the developer will choose technology B. On the other hand, in the case in which the customer does not have negotiation power, the developer should choose technology A to maximize the expected profit.

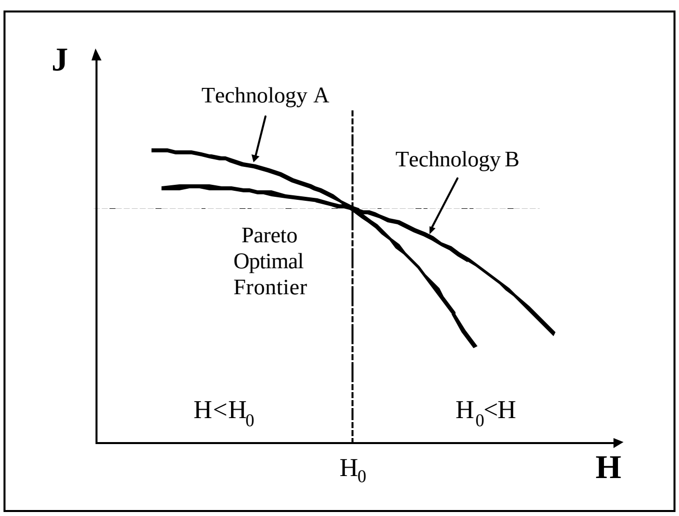

Figure 5: Pareto optimal frontiers of Solution 2

Solution 3

The Pareto optimal frontiers of the two technology choices may cross at more than one point as shown in Figure 6. In this case, the decision cannot be made easily. If the required consumer's surplus can be identified, the developer can choose the best technology choice at the required expected consumer's surplus level. In the case in which there are too many intersections and the required H cannot be defined, the developer may use other logical decision methods to choose the right technology. For example, if the Maximin method (maximizing the minimum value) is used, the developer will choose technology A. This is because the minimum expected profit of technology A is higher than that of the technology B.

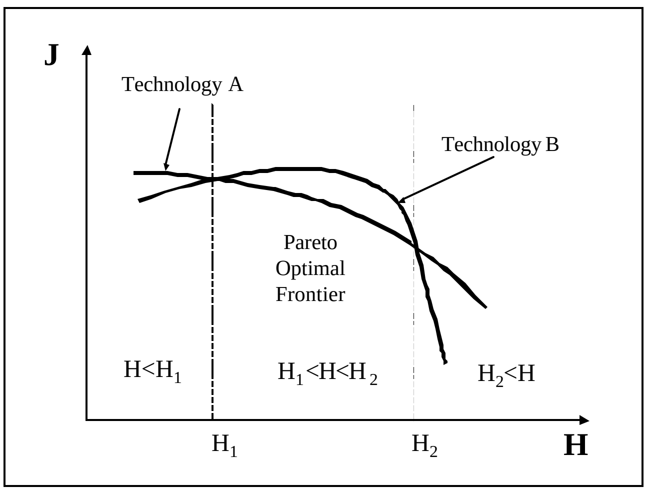

Figure 6: Pareto optimal frontiers of Solution 3

6 CALCULATION OF LOAD-POINT RELIABILITY WITH THE LOAD PRIORITY METHOD

6.1 Introduction

Considering the investment options that the developer may face, one of the most interesting options is the development of a generation portfolio, a set of generators with different technologies, to serve a group of customers. In this case, the generators together can be considered as a generation pool. The generation resources are allocated to individual customers according to certain allocation criteria.

To formulate the profit equation of the developer in this case requires not only generator reliability, but also load-point reliability. Determining the load-point reliability is not a simple calculation because there is no one-to-one relationship between generator reliability and load-point reliability. The load priority method has been developed for the calculation of load-point reliability.

To simplify the calculation, it is assumed that, in a typical distributed generation project, the number of generators is not large. The number of generation technologies at a project site is also limited. Therefore, the computation is not a major problem. The generators are located close to the loads. The distribution wires are short and therefore neglected.

6.2 The Load Priority Method

The load priority method has been developed for the calculation of load-point reliability based on load priority. In other words, the allocation of generation resources is based on the priorities of the customers. Normally, when the customers agree to sign the purchase contract with the developer, the load priority has to be defined in the contract so that load curtailments can be made in the event of generator failure. The developer can use a load management system, such as the load shedding system, to control the operation of the generation portfolio.

Mathematically, the calculation of load-point reliability has to be incorporated into the developer's decision criteria in the form of optimization constraints. The generation installed capacities, the reliability of the generators and the customer load information are all required for the calculation. However, in the optimization process, the installed capacities and generator reliabilities, which are decision variables, are not predetermined.

In order to explain the methodology, the discussion is divided into two parts. First, a generation portfolio with predetermined capacities and reliabilities is assumed. Load-point reliabilities are calculated from the predetermined information. Second, the methodology discussed in the first step is converted into optimization constraints.

6.2.1 Part I: Predetermined Generator Information

The methodology is developed from the following case. A generation portfolio with three generators (represented by G_1, G_2, and G_3) supplies real power to five customers (represented by

C_1, C_2, C_3, C_4, and C_5). The procedure consists of five steps:

1. Defining the generation capacities and reliabilities of the generators in the portfolio
2. Determining all the possible generator failure events and calculating the probability and available capacity of each event
3. Assigning a priority number to every customer load and formulating the load priority table
4. Matching the load priority table with each generator failure event to determine curtailed loads
5. Calculating the load-point reliability of each customer

Step 1

The installed capacities and reliabilities of the generators in the generation portfolio are defined as shown in Table 2.

Table 2: Information of the generation portfolio

Generation Unit	**Installed Capacity (kW)**		**Reliability (R_j)**	
G_1	K_1	100	R_1	0.95
G_2	K_2	150	R_2	0.90
G_3	K_3	250	R_3	0.92
	Total	**500**		

Step 2

Considering the reliability of the generation portfolio, there are seven generator failure events and one normal event, in which none of the generators fails. All the possible events are listed in Table 3. Columns three and four of the table show the probability and the available capacity when each event occurs.

Step 3

Each customer is assigned a priority number. The customer with the highest priority, which is represented by the lowest priority number (priority 1), is curtailed the last in the event of a supply shortage. However, the higher priority customers have to pay higher prices. The load priorities and prices are defined in the bilateral agreement between the developer and the customers. Table 4 shows the load information.

Table 3: List of generation failure events

Event	**Failed Generator**	**Probability**		**Available Capacity (kW)**	
E_1	G_1 G_2 and G_3	$p_1 = (1-R_1)(1-R_2)(1-R_3)$	0.0004	$AC_1 = 0$	0
E_2	G_1 and G_2	$p_2 = (1-R_1)(1-R_2) - p_1$	0.0046	$AC_2 = K_3$	250
E_3	G_1 and G_3	$p_3 = (1-R_1)(1-R_3) - p_1$	0.0036	$AC_3 = K_2$	150
E_4	G_2 and G_3	$p_4 = (1-R_2)(1-R_3) - p_1$	0.0076	$AC_4 = K_1$	100
E_5	G_1 only	$p_5 = (1-R_1) - p_1 - p_2 - p_3$	0.0414	$AC_5 = K_2 + K_3$	400
E_6	G_2 only	$p_6 = (1-R_2) - p_1 - p_2 - p_4$	0.0874	$AC_6 = K_1 + K_3$	350
E_7	G_3 only	$p_7 = (1-R_3) - p_1 - p_3 - p_4$	0.0684	$AC_7 = K_1 + K_2$	250
E_8	None	$p_8 = (1 - \sum_{i=1}^{7} p_i)$	0.7866	$AC_8 = K_1 + K_2 + K_3$	500

Table 4: Load Priority Table

Customer	**Priority**	**Load (kW)**		**Cumulative Load**
C_1	1	Q_1	100	100
C_2	2	Q_2	150	250
C_3	3	Q_3	100	350
C_4	4	Q_4	50	400
C_5	5	Q_5	50	500

Step 4

During a supply shortage period, the loads are curtailed according to priority. The curtailment is determined by comparing the load priority table with the available capacity. For example, if event E_5, in which generator G_1 fails and generators G_2 and G_3 are in operation, happens, the available capacity is limited to 400 kW. Customer C_5 has to be curtailed to preserve the higher priority customers. Table 5 and Table 6 show the connected and curtailed loads in all events listed by event and customer respectively.

Table 5: Load curtailment table listed by event

Event	**Probability**	**Available Capacity**	**Connected Load**	**Curtailed Load**
E_1	0.0004	0	-	C_1, C_2, C_3, C_4, C_5
E_2	0.0046	250	C_1, C_2	C_3, C_4, C_5
E_3	0.0036	150	C_1	C_2, C_3, C_4, C_5
E_4	0.0076	100	C_1	C_2, C_3, C_4, C_5
E_5	0.0414	400	C_1, C_2, C_3, C_4	C_5
E_6	0.0874	350	C_1, C_2, C_3	C_4, C_5
E_7	0.0684	250	C_1, C_2	C_3, C_4, C_5
E_8	0.7866	500	C_1, C_2, C_3, C_4, C_5	-

Table 6: Load curtailment table listed by customer

Customer	**Load (kW)**	**Load Connection Event**	**Load Curtailment Event**
C_1	100	E_2, E_3, E_4, E_5, E_6, E_7, E_8	E_1
C_2	150	E_2, E_5, E_6, E_7, E_8	E_1, E_3, E_4
C_3	100	E_5, E_6, E_8	E_1, E_2, E_3, E_4, E_7
C_4	50	E_5, E_8	E_1, E_2, E_3, E_4, E_6, E_7
C_5	50	E_8	E_1, E_2, E_3, E_4, E_5, E_6, E_7

Step 5

The last step is calculating the load-point reliability by summing the probabilities of connected events for each customer. As shown in Table 7, the customer with a higher load priority has higher load-point reliability.

Table 7: Load-point reliability

Customer	Priority	Load-point Reliability	
C_1	1	$R_1 = p_2 + p_3 + p_4 + p_5 + p_6 + p_7 + p_8$	0.9996
C_2	2	$R_2 = p_2 + p_5 + p_6 + p_7 + p_8$	0.9885
C_3	3	$R_3 = p_5 + p_6 + p_8$	0.9155
C_4	4	$R_4 = p_5 + p_8$	0.8280
C_5	5	$R_5 = p_8$	0.7866

6.2.2 Part II: Optimization Constraints

The methodology discussed in the previous section assumes that the information about the generators in the portfolio is predetermined. However, in the optimization process, the information is not available because installed capacities and reliabilities are decision variables that the developer has to choose to maximize the profits. The methodology is modified by incorporating the calculation of the load-point reliability into the optimization constraints. The procedures discussed in the previous section are converted into the constraint equations.

Step 1

The information about the generators in the portfolio is defined by the following variables:

K_j = Installed capacity of generator j

R_j = Reliability of generator j

n = Number of generators in the portfolio

Step 2

The calculations of the probability and available capacity of each possible event are complicated. The following terms are defined for the calculations.

E_{ab} = Generator-failure event a and sub-event b

Θ = Set of total generators in the generation portfolio, defined by:

$$\Theta = \{G_1, G_2, ..., G_j, ..., G_n\}$$

Ψ_{ab} = Set of failed generators in the event E_{ab}

Φ = Empty set

p_{ab} = Probability of the event E_{ab}

$P_{xy}(\Psi_{ab})$ = Function defined by:

$$\mathbf{P_{xy}}(\Psi_{ab}) = p_{xy} \text{ if } \Psi_{xy} \cap \Psi_{ab} \neq \Phi, \text{otherwise } 0$$

AC_{ab} = Available capacity of the event E_{ab}

Using the concept developed in the previous section, the probabilities and available capacities of all generator failure events can be calculated as shown in Exhibit 1.

Step 3

The load priority table is defined as shown in Table 8. In the load priority table, the customers are listed according to load priority so that the cumulative load (QC) can be summed up. It is assumed that there are m customers who sign purchase contracts.

Table 8: Load Priority Table

Customer	**Priority**	**Load (kW)**	**Cumulative Load**
C_1	1	Q_1	$QC_1 = \sum_{i=1}^{1} Q_i$
C_2	2	Q_2	$QC_2 = \sum_{i=1}^{2} Q_i$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
C_k	k	Q_k	$QC_k = \sum_{i=1}^{k} Q_i$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
C_m	m	Q_m	$QC_m = \sum_{i=1}^{m} Q_i$

Step 4

The customer is curtailed in the event that the available capacity is less than the cumulative load at the customer's load priority. On the other hand, the customer connects to the system when the available capacity is equal to or more than the cumulative load at the customer's load priority. Mathematically, the probability of load connection in the event E_{ab} is determined by the following function:

$$L_{i,ab} = \mathbf{L}(C_i, E_{ab}) = p_{ab}, \text{if } AC_{ab} \geq QC_i, \text{otherwise } 0 \quad \text{(A.1)}$$

Step 5

Finally, the load-point reliability of the customer is calculated by:

$$R_i = \sum_{x=1}^{a} \sum_{y=1}^{b} L_{i,ab} \quad \text{(A.2)}$$

In conclusion, the equations formulated in steps 2, 3, 4, and 5 are incorporated into the optimization process in the form of constraints.

7 Conclusion

The decision criteria of two decision makers, developers and end-use customers, under various scenarios are discussed. The objective function of the developer is maximizing expected profit discounted by risk factor. For end-use customers, the objective functions is maximizing expected consumers' surplus discounted by risk factor. This paper introduced a concept of coupled decision criteria from the developers' perspective. Load priority method is introduced for the calculation of load-point reliability.

Reference

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EXHIBIT 1: Probabilities of the generator failure events

Event Index (a)	**# failed generators**	**Sub-event Index (b)**	**Event**	**Failed Generator**	**Available Capacity (AC_{ab})**	**Probability (p_{ab})**
1	n	1	E_{1n}	$\Psi_{11} = \Theta$	$AC_{11} = 0$	$p_{11} = \prod_{j=1}^{n} (1 - R_j)$
⋮	⋮	⋮	⋮	⋮	⋮	⋮
k	k	1	E_{k1}	$\Psi_{k1} = \Theta - \left\{ \begin{matrix} G_{k+1}, G_{K+2}, \ldots, \\ G_{n-1}, G_n \end{matrix} \right\}$	$AC_{k1} = \sum_{j=1}^{n-k} K_j$	$p_{k1} = \prod_{j=k+1}^{n} (1 - R_j) - \sum_{x=1}^{k-1} \sum_{y=1}^{b} P_{xy}(\Psi_{k1})$
		⋮	⋮	⋮	⋮	⋮
		$b = \frac{n!}{(n-k)!k!}$	E_{kb}	$\Psi_{kb} = \Theta - \left\{ \begin{matrix} G_1, G_2, \ldots, \\ G_{k-1}, G_k \end{matrix} \right\}$	$AC_{kb} = \sum_{j=k+1}^{n} K_j$	$p_{kb} = \prod_{j=1}^{k} (1 - R_j) - \sum_{x=1}^{k-1} \sum_{y=1}^{b} P_{xy}(\Psi_{kb})$
⋮	⋮	⋮	⋮	⋮	⋮	⋮
n	1	1	E_{n1}	$\Psi_{n1} = \{G_n\}$	$AC_{n1} = \sum_{j=1}^{n-1} K_j$	$p_{n1} = (1 - R_n) - \sum_{x=1}^{n-1} \sum_{y=1}^{b} P_{xy}(\Psi_{n1})$
		⋮	⋮	⋮	⋮	⋮
		n	E_{nn}	$\Psi_{nn} = \{G_1\}$	$AC_{nn} = \sum_{j=2}^{n} K_j$	$p_{nn} = (1 - R_1) - \sum_{x=1}^{n-1} \sum_{y=1}^{b} P_{xy}(\Psi_{nn})$
$n+1$	0	1	$E_{(n+1)1}$	$\Psi_{(n+1)1} = \{\ \}$	$AC_{(n+1)1} = \sum_{j=1}^{n} K_j$	$p_{(n+1)1} = \left(1 - \sum_{x=1}^{n} \sum_{y=1}^{b} p_{xy}\right)$