MIT EL 00-006

Energy Laboratory

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Valuation of Generation Assets with Unit Commitment Constraints under Uncertain Fuel Prices

November 2000

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Energy Laboratory Publication # MIT_EL 00-006

November 2000

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Abstract

Market based valuation of generation assets is a critical problem in competitive power markets. Previous approaches to this problem has included modeling the cash flow from a generator as a spread option between electricity and fuel prices, which ignores the effect of flexibility and startup costs. Simulation based approaches with full unit commitment constraint have been presented, but they are computationally extremely demanding.

In this report, a principal component based model for electricity prices is applied. It is shown how this approach can reduce the unit commitment problem to two stage dynamic programming (DP) problem. The information gained by solving the DP for possible electricity and fuel price states, is stored in a lookup table. This table is used to map simulated fuel and electricity price states to generator cash flow. As a result we are able to perform simulation based valuation of generators over multi-year periods, with minimal computational complexity.

Introduction

As the electric utility industry becomes more competitive, the question of how to value generation assets becomes critical. This problem is typically approached by defining the generator in terms of its efficiency (heat rate), in converting fuel to electricity. Based on this rating, the valuation is performed by modeling the generator as a spread option between the price of the fuel used and the price of electricity [Deng, Johnson, Sogomonian (1998)]. The payoff from such an option is given by,

$$CF_k = \max\left\{P_k^e - C(P_k^f),0\right\}$$

where P_k^e is the price of electricity and C is the cost marginal cost of production as a function of fuel price P_k^f.

The problem with this formulation is that it ignores several important constraints involved in the operation of the unit, such as start-up and shut down costs, minimum run time, and maximum ramp rate [Tseng, Braz (1998)] . These constraints have a significant effect on how units are bid into, and dispatched by, a spot market operator, and therefore on the owners cash flow. By ignoring the unit commitment constraints, one is likely to undervalue plants with significant flexibility (such as micro turbines and fuel cells) while overvaluing large inflexible fossil plants.

The reason why the unit commitment problem is often ignored in the valuation of a power plant can be linked to computational complexity. In general the operator of the unit has to solve a complex dynamic programming problem to arrive at the optimal unit commitment decision for the generator [Eric, Ilic (1999)]. This is a computationally intensive problem with polynomial growth over the time horizon over which the optimization is carried out. Therefore while it is feasible to solve the unit commitment decision for a day ahead bidding problem [Tseng, Braz (1998)], it is extremely challenging to extend this notion to a multi-year valuation problem.

In this working paper, we propose a new method for valuing generation assets with unit commitment constraints, using a principal component based model for spot price described in details in [Skantze, Gubina, Ilic (2000)]. The effectiveness of the principal component representation comes from being able to determine the hourly prices within each day. This is qualitatively different from using a single daily spot price, which does not recognize intra-day price variations. By applying principal components, we are able to define the today's net profit from the generation asset as a function only of today's and yesterday's average spot price. By storing the mapping from the state of the spot price to the cash flow of the generator in a lookup table, we are able to simulate generator profits over multiyear periods with minimal computational complexity.

Next we introduce a stochastic model for the fuel price. This adds a third dimension to our lookup table, but still allows us to simulate the cash flow for the generator with a computational time growing linearly with the length of the valuation period.

The principal component based model is applied next to the problem of hedging generation assets. By defining the daily cash flows from the unit as a derivate of fuel and electricity prices, we can derive optimal strategies for trading in fuel and electricity forward markets in order to manage generation risk.

A Principal Component Based Price Model for Electricity Spot Markets

The price model used is a simplified version of the bid based price model introduced in [Skantze, Gubina (2000)]. We define a daily [24*1] price vector P^e_d, whose elements are the 24 hourly electricity prices. Next we define the log of the price vector to be the sum of a deterministic and stochastic component. The deterministic component is composed of a monthly vector μ_m, which captures the seasonal characteristics of the electricity spot price. The stochastic component is modeled as the product of the principal component vector v_m, and a daily stochastic scalar weight w_d. The principal component captures the shape of daily price variations from the seasonal mean μ_m, while the weight describe the magnitude of the deviation as well as its correlation over time. The log of the vector of spot market prices can be written as:

$$\ln(P^e_d) = \mu^e_m + w^e_d v^e_m$$

Next we model the process describing the evolution of the weights w^e_d. We choose a two-factor discrete time mean reverting process. This captures important features of electricity markets such as short-term mean reversion and long term stochastic growth. For an in-depth discussion of modeling electricity prices, please see [Skantze, Gubina (2000)].

$$e^e_{d+1} - e^e_d = -a^e(e^e_d) + s^e_m z^e_d$$

$$d^e_{d+1} - d^e_d = ?^e + s^{de}_m z^{de}_d$$

Where,

$$e^e_d = w^e_d - d^e_d$$

The form of the price process postulates that hourly spot prices will be log-normally distributed. Furthermore prices inside of a day are perfectly correlated, since they are a function of a single daily random variable w_d. This reduction in complexity is made possible by choosing the principal component in an intelligent manner.

Formulating the Unit Commitment Decision

To calculate cash flow in a one-day period, firstly a generator solves a unit commitment problem in order to determine when a unit is turned on or off optimally. The one-day cash flow is the expected sum of profits from operating in each hour. Let $CF_d(x_0)$ be cash flow at the d day,

$$CF_d(x_0) = \max_p E\{\sum_{k=0}^{24-1} P^e_{d,k} q - c_k(q)\}$$

Rewrite this optimization in a Dynamic Programming (DP) framework, adopted from Eric, Ilic (1998) as the following:

$$CF_d(x_0) = J_0(x_0) = \max_p E\{\sum_{k=0}^{23} u_k(P^e_{d,k} q_k - c_k(q_k, P^f_{d.k}) - I(x_k < 0)S) + (1-u_k)\cdot(c_f + I(x_k > 0)T\}$$

$$J_N(x_N) = 0$$

Reward-to-go in hour k:

$$J_k(x_k) = \max_{u_k} E\{u_k(P^e_{d,k}q_k - c_k(q_k, P^e_{d,k}) - I(x_k < 0)S) + (1-u_k)\cdot(c_f + I(x_k > 0)T) + J_{k+1}(x_{k+1})\}$$

$$x_{k+1} = \begin{cases} \max(1,\ x_k + 1) & u_k = 1 \\ \min(-1, x_k - 1) & u_k = 0 \end{cases}$$

$$u_k \ge I(1 \le x_k \le t_{up})$$

$$u_k \ge 1 - I(-t_{dn} \le x_k \le -1)$$

$$q_{min} \le q_k \le q_{max}$$

$$I('TRUE') = 1$$

$$I('FALSE') = 0$$

The optimal policy is applied to obtain the maximum expected profits. The above problem is a full-blown version in which there are several sources of uncertainties. The first one is from electricity spot prices s^e_m, and the second one is from fuel prices.

Approximation of Price Model Used in Unit Commitment

When solving the unit commitment bidding problem, we use an approximate version of the price process. In the full-blown model we can write next days weight as a function of today's states:

$$w^e_{d+1} = (1-a^e)\, w^e_d + a^e d^e_d + ?^e + s^e z^e_d + s^{de} z^{de}_d .$$

We here assume that $a^e << 1$, and $s^e >> s^{de}$. We only use this assumption when formulating the day ahead bidding strategies. With this assumption we can write the weight process as:

$$w^e_{d+1} = w^e_d + ?^e + s^e z^e_d .$$

This effectively states that in the very short term (day ahead) we can ignore the mean reversion as well as the long term volatility. It should be noted that we only use this assumption to arrive at a bidding strategy. When simulating future spot price for valuation purposes we use the full blown version of the price model.

Therefore, we can simplify the above unit-commitment by assuming that

1) $q_k = q_{max}$,
2) in a one-day period, $P^f_{d.k}$ is given $\forall k$,
3) $a^e = 0$, during a one-day period,
   $\tilde{w}_{d+1} = \tilde{w}_d + s^e_m z^e_d$

Therefore,

$$J_k(x_k, w_{d-1}, a^e) = \max_{u_k} E_{\tilde{w}_d|\tilde{w}_{d-1}}\{u_k(P^e_{d,k}q_k - c_k(q_k) - I(x_k < 0)S) + (1-u_k)\cdot(c_f + I(x_k > 0)T)$$

$$+ J_{k+1;}(x_{k+1}, w_{d-1}, a^e)\}$$

and,

$$CF_d(x_0, \tilde{w}_{d-1}, a^e) = J_0(x_{0,} \tilde{w}_{d-1}, a^e)$$

Creating a Lookup Table of Cash flows

We have shown how to calculate cash flow and a on/off policy for a generator in a given day d. This cash flow is an expected cash flow, given $\tilde{w}_d = \tilde{w}_{d,i}$. $\tilde{w}_{d,i}$ is a sample value of $\tilde{w}_d$, which is continuously normally distributed with $(\tilde{w}_{d-1} + ?^e)$ mean and standard deviation s^e_m. To create a lookup table mapping a pair $(\tilde{w}_{d,j}, \tilde{w}_{d-1,i})$ to a cash flow, we generate $\tilde{w}_{d,j}$ given $\tilde{w}_{d-1,i}$; and apply the optimal policy $p(\tilde{w}_{d-1,i})$ to determine the cash flow in the d period with $\tilde{w}_{d,j}$, or $P^e_{d,j}$. Therefore, in period d, we have

$$CF_d(\tilde{w}_{d-1.i}, \tilde{w}_{d,j})$$

$$= \sum_{k=0}^{23} \{u_k(\tilde{w}_{d-1,i})(P^e_{d,k}(\tilde{w}_{d,j}) \cdot q - c(q) - I(x_k < 0)S) + (1 - u_k(\tilde{w}_{d-1,i})) \cdot (c_f + I(x_k < 0)T)\}, \forall j$$

Repeating this process using $\tilde{w}_{d-1,i}$ for other i, one obtains a cash flow matrix which an element (i,j) is a cash flow associating with both $\tilde{w}_{d,j}$ and $\tilde{w}_{d-1,i}$. This matrix captures possible cash flows in a given month m with a simplified price process of each day d.

$$\overline{CF}_m = \begin{bmatrix} CF_d(\tilde{w}_1, \tilde{w}_1) & CF_d(\tilde{w}_1, \tilde{w}_2) & \cdots & CF_d(\tilde{w}_1, \tilde{w}_N) \\ & \ddots & & \\ \vdots & & CF_d(\tilde{w}_i, \tilde{w}_j) & \vdots \\ & & \ddots & \\ CF_d(\tilde{w}_N, \tilde{w}_1) & & \cdots & CF_d(\tilde{w}_N, \tilde{w}_N) \end{bmatrix}$$

For each month m, a cash flow matrix can be calculated by using in the same method. Note that each cash flow matrix is obtained by assuming a constant fuel price $P^f_{d,k}$ for all k and d within each month m.

$$\{\overline{CF}_1, \overline{CF}_2, \ldots, \overline{CF}_{12} \mid P^f_{d,k} = ?^f_l\}$$

Incorporating Stochastic Fuel Prices

Next we propose a model which will allow us to include stochastic fuel prices. We here assume that we are dealing with a gas-fired plant, but the model is equally applicable to oil or coal. Since gas is a storable commodity, it experiences less short-term volatility, and very little intra-day volatility. We will therefore make the following assumptions.

1. There is only as single daily gas price $P^f_{d,k}$.
2. In the unit commitment decision, the day ahead gas price is assumed to be forecasted with near-perfect accuracy (i.e. assumed to be deterministic).

Next we postulate a model for the daily gas price. The log of the price is written as the sum of a deterministic seasonal and a stochastic component.

$$\ln(P^f_d) = \mu^g_m + w^e_d$$

Note that we do not need to apply the principal component approach since the price is a scalar. The stochastic component is described by a two factor mean reverting model.

$$e^{f}_{d+1} - e^{f}_{d} = -a^{f}(e^{f}_{d}) + s^{f}_{m} z^{f}_{d}$$

$$d^{f}_{d+1} - d^{f}_{d} = ?^{f} + s^{d\,f}_{m} z^{d\,f}_{d}$$

where,

$$e^{f}_{d} = w^{f}_{d} - d^{f}_{d}$$

To apply this model we first need to expand the lookup table to include gas price as a third dimension. Note that the assumption that unit commitment takes the day ahead gas price as deterministic allows us to add only one rather than two dimensions to the lookup table.

We now generate simulated paths for future w's for electricity and gas. The lookup table converts this into paths of future cash flows.

To capture the dynamic of fuel prices which is assumed to vary on a monthly basis, we create a set of cash flow matrices with different fuel prices, obtaining

$$\begin{cases} \{\overline{CF}_{1,}\overline{CF}_{2,}\ldots,\overline{CF}_{12} \mid P^{f}_{d,k} = ?^{f}_{1}\} \\ \vdots \\ \{\overline{CF}_{1,}\overline{CF}_{2,}\ldots,\overline{CF}_{12} \mid P^{f}_{d,k} = ?^{f}_{L}\} \end{cases}$$

Note that we can generate N samples of w of fuel prices to create an $(N \times N \times N)$ cash flow matrix for each month m. This matrix completely captures uncertainty due to both electricity and fuel prices.

Linking Simulated Prices to the Lookup Table to Generate Simulated Cash Flows

Once the lookup table has been created, we can use the full blown price model to generate simulated weights. The lookup table is then used to generate a path of cash flows from a path of weights. The simulation time is linear in the length of the valuation period. Furthermore we are not restricted to the proposed model for generating weights. The lookup table can be linked to any stochastic model which produces weights for the principal components.

Generation Asset Valuation

Value of a generator V is an expected sum of discounted cash flows during the period of valuation.

$$V = E\{\sum_{d=0}^{D}(r)^{d}\cdot CF_d(w_d^e)\},$$

Where r is a discounted factor, in which $0 < r < 1$, and D is a period of valuation (such as a 15-year period or a 15*365-day period).

There are two valuation methods that we consider here:

1) A Monte Carlo Simulation Method

   a) *M* paths of (w^e, w^d) are generated.

   $$\begin{bmatrix} \overline{w}^e \\ \overline{w}^g \end{bmatrix}^i = \begin{bmatrix} w_1^e, w_2^e, \ldots, w_N^e \\ w_1^f, w_2^f, \ldots, w_N^f \end{bmatrix}^i;$$

   b) For each path i of $[w^e, w^f]^i$, each cash flow is obtained by choosing a cash flow associated with each pair of $[w_j^e, w_j^f]^i$ from the lookup table. The value of a generator if w_j^e and w_j^e follow path *i* is the sum of discounted cash flows.

   $$V^i = \sum_{d=0}^{D}(r)^d \cdot CF_d^i(w_d^e, w_d^f)$$

   c) Value of a generator is equal to

   $$V = \frac{1}{M}\{\sum_{i=1}^{M} V^i\} = \frac{1}{M}\{\sum_{i=1}^{D}(\sum(r)^d \cdot CF_d^i(w_d^e, w_d^f))\}$$

2) A Multinomial Tree Method

Instead of using Monte Carlo simulation, we can use a multinomial tree method. The simplest version of the tree method is a binomial tree one. A binomial tree can be used in the case one source of uncertainty exists. For example, if gas prices are known at any time d, the only uncertainty in generation asset valuation comes from electricity prices. Each node on a tree associated with a day d. At the end of each day d, $w_d^{e,i}$ either goes up with probability p to be $w_{d+1}^{e,i+1}$ or goes down with probability 1-p to be $w_{d+1}^{e,i-1}$. Hence, at each node a cash flow associated with w_d^{e} ($w_d^{e,i+1}$ or $w_d^{e,i-1}$) conditioned on the previous node $w_{d-1}^{e,i}$ is simply obtained from the lookup table. We expand the tree from a single node on day 0 to 2^N nodes on day N. The value at each node i on day d V_d^i is the expected sum of discounted cash flows of the next adjacent nodes, plus the cash flow associated with $w_d^{e,i}$ incurred at that node.

$$V_d^i = CF_d^i(w_{d-1}^{e,j}, w_d^{e,i}) + r(p \cdot CF_{d+1}^{i+1} + (1-p) \cdot CF_{d+1}^{i-1}), j = \{i\text{-}1, i+1\}$$

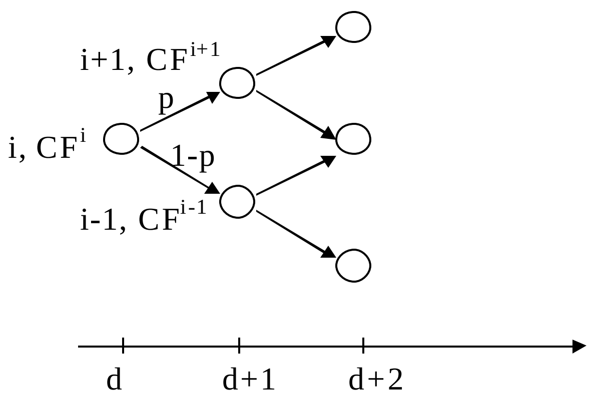

Figure 1. A Binomial-Tree Representation

For a case of more than one sources of uncertainty, two additional branches will be added to capture each additional source of uncertainty. This will make the problem become more complex since nodes grow exponentially with time. A Monte Carlo approach might be more applicable to deal with more than one sources of uncertainty.

Conclusion

As shown in this report, the day-ahead process of deciding on an optimal commitment strategy for generation assets with unit commitment constraints under uncertain fuel prices, is an extremely complex problem. While in theory one can value a unit by simply extending this commitment decision for a multiyear period, by simulating a rang of possible fuel and electricity price paths, this approach is extremely computationally demanding. The use of principal component theory in price modeling recognized that there are dominant patterns in the hourly price deviations within a day. These patterns can be exploited to reduce the number of random variables in the unit commitment decision, and greatly reduce the computational complexity of the problem. We have illustrated how we can characterize the day ahead bidding strategy of a generator as a function of three states, representing electricity and fuel prices. This leads to the creation of a lookup table which effectively stores all information relating to the unit commitment problem. Ones this lookup table is created, the problem of valuing the generator is trivial, since all we have to do is to generate simulated price paths for fuel and electricity prices. The approach is extremely computationally efficient and allows us to simulate the value of generation assets over multi-year period.

The contribution of this report is to present a computationally feasible valuation method, which will allow users to differentiate between technologies based on flexibility as well as fuel efficiency. The authors believe that this will have significant impact on the investment choice in the new industry both on a whole-sale and distribution level.

Appendix:

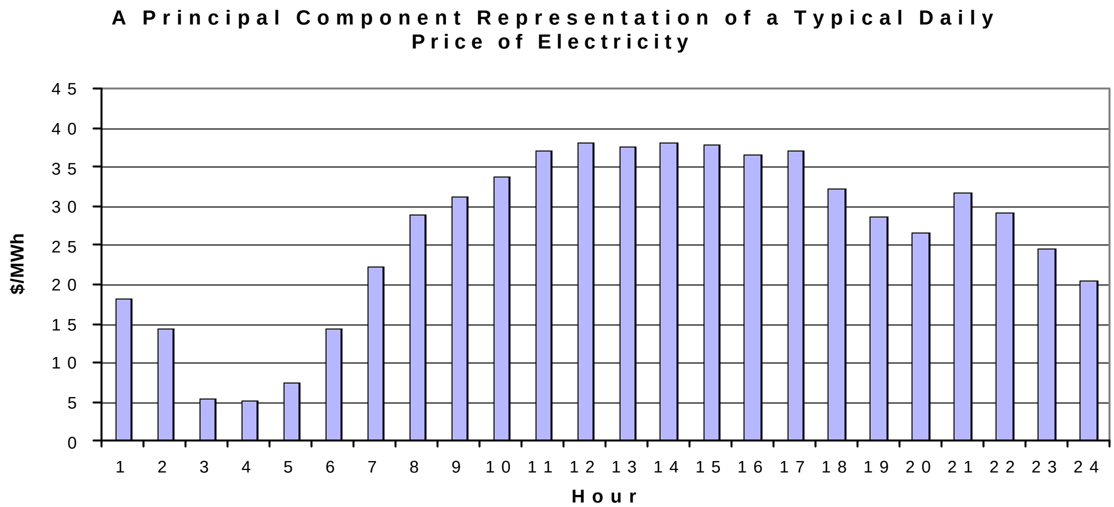

Figure 1. Principal Component Representation of a Typical Daily Price of Electricity[1]

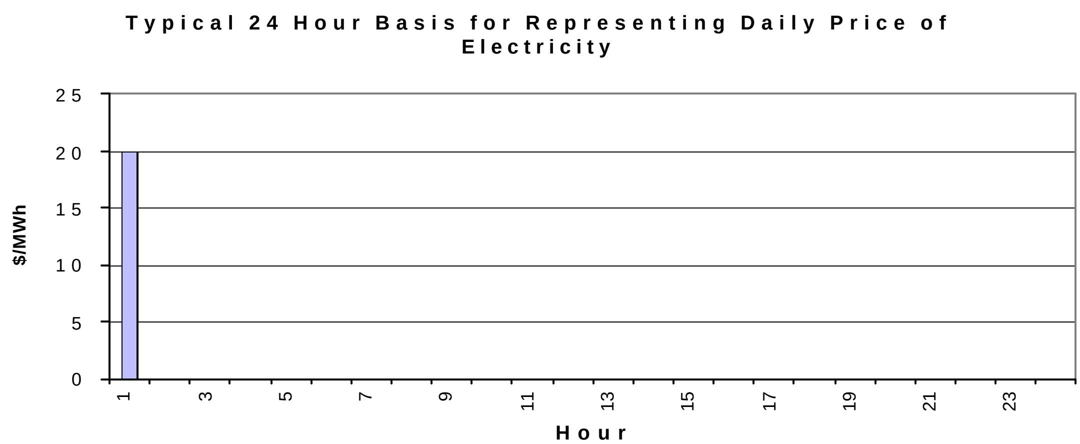

Figure 2a. A Typical 24 Hour Basis for Representing Daily Price of Electricity (Hour 1).

[1] Data is obtained from Petter and Gubina (2000).

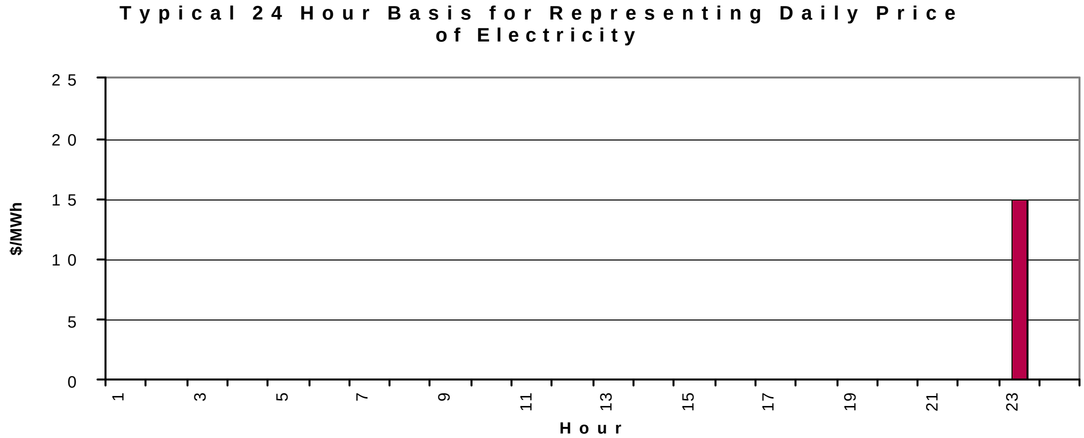

Figure 2b. A Typical 24 Hour Basis for Representing Daily Price of Electricity (Hour 23).

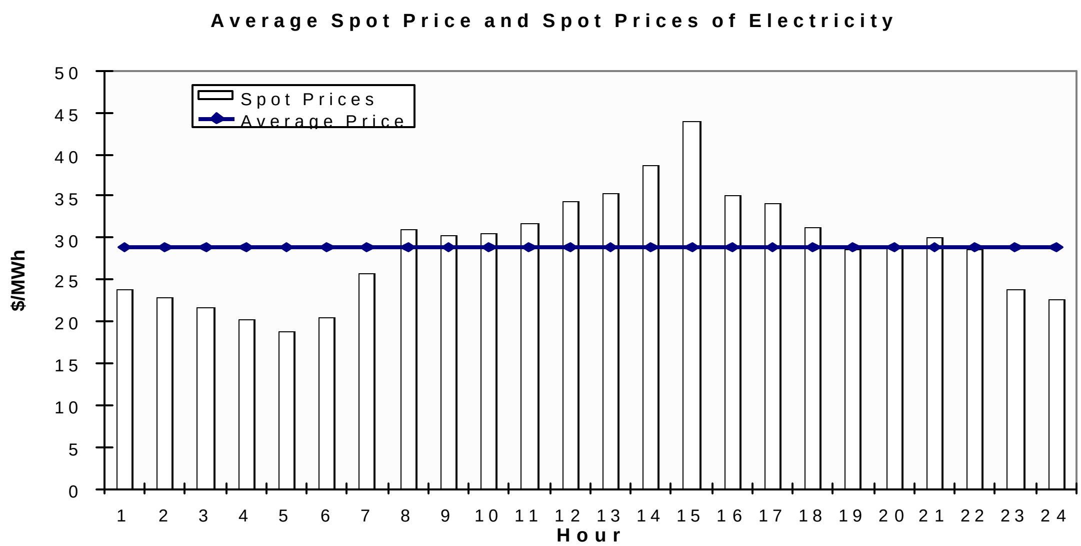

Figure 3 Average Spot Price of Electricity and Spot Prices of Electricity

Reference:

[1] Deng, S., Johnson B., Sogomonian, A., (1998) 'Exotic Electricity Options and the Valuation of Electricity Generation and Transmission Assets,' presented at the Chicago Risk Management Conference, Chicago, May 1998.

[2] Tseng, C-L., Barz, G., (1998), 'Short-Term Generation Asset Valuation,' Proceedings of the Thirty-second Annual Hawaii International Conference on System Sciences, http://www.computer.org/proceedings/hicss/0001/00013/00013028abs.htm

[3] Allen, E., Ilic, M.D., (1999), *Price-Based Commitment Decisions in the Electricity Market*, Springer-Verlag, London.

[4] Hull, J.: Options, Futures and other Derivatives, 4^{th} ed., Prentice-Hall, 1999, ISBN 0-13-022444-8.

[5] Skantze, P., Gubina, A., Ilic, M., "Bid-based Stochastic Model for Electricity Prices: The Impact of Fundamental Drivers on Market Dynamics", MIT Energy Laboratory Technical Report EL 00-004, November 2000.

[6] Schwartz, E.: "The Stochastic Behavior of Commodity Prices: Implications for Valuation and Hedging", *The Journal of Finance*, Vol. 7, No. 3, July 1997.

[7] Bertsekas, D. P., Dynamic Programming: Deterministic and Stochastic Models, Prentice-Hall 1987.